DELAWARE VALLEY SCHOOL DISTRICT

PLANNED INSTRUCTION

A PLANNED COURSE FOR:

ADVANCED TOPICS (2^{nd} SEMESTER)

Grade: 12

Date of Board Approval: ___2014_______________

DELAWARE VALLEY SCHOOL DISTRICT

PLANNED INSTRUCTION

Title of Planned Instruction: Advanced Topics (2^{nd} semester)

Subject Area: Mathematics Grade Level: 12

Course Description:

This course is designed for advanced mathematics students to be taken in conjunction with Advanced Placement Calculus (either AB or BC). Advanced Topics (1^{st} semester) is not a prerequisite for this course. Advanced Topics (2^{nd} semester) focuses on branches and applications of advanced mathematics such as calculus, combinatorics, probability, matrices, linear programming, cryptography, graph theory, and topology. Also, the use of technology, such as the TI-84 and TI-89 graphing calculators, internet websites, and Derive software, will be emphasized. The course will also emphasize development of students' mathematical problem solving skills in a variety of areas including algebra, geometry, probability, logic, and word problems in order to be successful in mathematics contests at the local, state, and national levels. Furthermore, throughout the course students' proper use of terminology and precise language both verbally and in writing will be stressed.

Time/Credit for the Course: 46 minutes per day for 1 semester/ ½ credit

Curriculum Writing Committee: Gary Dennis

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Resources for Advanced Topics (2^{nd} semester)

Calculus by Howard Anton

Cryptological Mathematics by Robert Edward Lewand

Euler's Gem by David S. Richeson

Foundations of Higher Mathematics by Peter Fletcher and C. Wayne Patty

Full House by Stephen Jay Gould

Graph Theory-Euler's Rich Legacy by Wayne Copes, Clifford Sloyer, Robert Stark, and William Sacco

Introduction to Derive for Windows by Bernhard Kutzler

Introduction to Operations Research by Frederick S. Hiller and Gerald J. Lieberman

Journey Through Genius – The Great Theorems of Mathematics by William Dunham

The Man Who Loved Only Numbers – The Story of Paul Erdös and the Search for Mathematical Truth by Paul Hoffman

The Mathematical Universe by William Dunham

Mathematics of Choice –How to Count Without Counting by Ivan Niven

"Mathematics of Distortion" Handout

Möbius Story Wind and Mr. Ug [www.keepvid.com].mp4

Precalculus by Michael Sullivan

"Why Mathematicians Now Care About Their Hair Color" by Sara Robinson

"Travelers' Dilemma" by Kaushik Basu

"Two Centuries On, a Cryptologist Cracks a Presidential Code" by Rachel Emma Silverman

"The River Crossing Game" from Mathematics Magazine, February, 2007

Problems and solutions from previous mathematics contests such as the Pennsylvania Mathematics League, the American Mathematics Competition, and the American Invitational Mathematics Exam.

TI-84 and TI-89 graphing calculators

"Derive" software

Teacher designed VisualBasic simulations of the Monty Hall problem, the Cooties problem, and Random Walks.

Websites: www.mathworld.com (various topics and sites)
www.combinatorics.net
www.ptri1.tripod.com
www.csam.montclair.edu/~kazimir/history.html
www.teacherlink.org/content/math/interactive/flash/home.html
www.indiana.edu/~minimal/archive/index.html
www.virtualmathmuseum.org
www.people.hofstra.edu/Stefan_Waner/RealWorld/Summary4.html
www.numberphile.com
http://www.oglethorpe.edu/faculty/~j_nardo/2000-2001/Spring%202001/great%20ideas/torusgames/index.html

Curriculum Map

Third Marking Period

Combinatorics

- Introductory problems
- Multiplication Principle of Counting
- Permutations
- Combinations
- Paths problem
- Number of orderings of objects in a circle
- Number of seatings of people under given restrictions
- Pigeonhole Principle and applications
- Inclusion-Exclusion Principle and applications
- Derangements
- Binomial expansions
- Pascal’s Triangle and applications
- Fibonacci sequence
- Applications of combinatorics

Probability

- Poker hands in a standard deck of cards
- Hat color problem
- Monty Hall Problem
- Travelers’ Dilemma
- Drug testing problem
- River Crossing Game

Fourth Marking Period

Markov Chains

- Definition of a Markov Chain
- Transition matrices, initial matrix, and analysis of Markov Chains
- Runner’s Shoes problem
- Cooties problem
- Lucky Charms problem
- Playground problem
- Expected value
- Random walks
- VisualBasic simulations for the Cooties problem and random walks

- Connections between the expected value of the Cooties problem, random walks, the theory of the disappearance of .400 baseball batting averages, and the theory of bacterial dominance

Matrices and Applications

- Arithmetic and algebra of matrices
- Inverse matrices
- Systems of linear equations
- Reduced row echelon form using a graphing calculator
- Linear programming
- Feasible regions, isoprofit lines, and solutions to linear programs
- Sensitivity analysis
- Simplex algorithm
- Cryptography

Graph Theory

- Basic terms and concepts
- Sprouts game
- Bridges of Königsberg
- Traceable graphs
- Chromatic number of a graph
- Applications of chromatic number
- Adjacency matrix of a graph
- Houses and Wells problem
- Planarity
- Four Color Map Theorem and its history
- Euler's Formula
- Connected graphs, cycles, and trees
- Minimal spanning trees
- Weighted graphs and applications

Topology

- Topologically equivalent figures
- Möbius strip and its properties and applications
- Topological properties of a torus and various knots
- Classic games played on a torus
- Klein bottles
- Minimal surfaces and their application to bubbles and soap films

Unit One: Combinatorics

Big Idea #1: Mathematical formulas and processes can provide efficient ways to count the number of ways that an event can occur.

Essential Questions:

- How can we count the number of ways that an event can occur without counting one by one?
- How can the formulas for combinatorics be applied to real world problems?

Concepts:

- Multiplication Principle of Counting
- Factorials
- Permutations
- Combinations
- Binomial expansion
- Pascal's Triangle
- Fibonacci Sequence
- Inclusion-Exclusion Principle
- Derangements
- Pigeonhole Principle
- Graphing Calculators
- Applications of combinatorics

Competencies:

- Apply the Multiplication Principle of Counting
- Evaluate and simplify factorial expressions
- Solve permutation problems
- Solve combination problems
- Expand binomials using the terms of Pascal's Triangle
- Find the number of paths on a grid
- Determine the number of arrangements of objects in a circle
- Determine the number ways to arrange people in a line with given restrictions
- Apply the Inclusion-Exclusion Principle
- Explain and apply the derangements formula
- Explain and apply the Pigeonhole Principle
- Analyze the Mutual Acquaintances/Strangers problem
- Find the terms of Pascal's Triangle using a graphing calculator
- Relate the terms of Pascal's Triangle to the terms of the Fibonacci Sequence
- Apply combinatorics techniques to real world problems

Unit Two: Probability

Big Idea #1: Mathematical formulas and processes can be used to solve various probability problems.

Essential Questions:

- How are probability problems solved?
- How can probability techniques and processes be applied to real world problems?

Concepts:

- Poker hands
- Hat Color Problem
- Monty Hall Problem
- Travelers' Dilemma Problem
- Drug Testing Problem
- River Crossing Game
- Markov Chains
- VisualBasic simulations
- Skewed Distributions

Competencies:

- Determine probability of possible poker hands in a standard deck of cards
- Analyze the Hat Color Problem
- Analyze the Monty Hall Problem
- Analyze the Travelers' Dilemma Problem
- Analyze the River Crossing Problem
- Explain the process of Markov Chains
- Apply Markov Chains to real world problems
- Analyze the various scenarios that lead to skewed distributions

Unit Three: Applications of Matrices

Big Idea #1: Arithmetic and algebra with matrices can be used to solve a variety of applied problems.

Essential Questions:
- What are the rules of arithmetic for matrices?
- How can matrix equations be solved?
- How can matrices be used to solve applied problems?

Concepts:
- Arithmetic of matrices
- Matrix equations
- Inverse of a matrix
- Systems of equations
- Linear Programming
- Cryptography

Competencies:
- Simplify matrix expressions using rules of arithmetic for matrices
- Solve matrix equations
- Find the inverse of a matrix
- Solve systems of equations using matrices
- Apply matrices to various cryptography methods
- Solve linear programming problems

Unit Four: Graph Theory and Topology

Big Idea #1: The concepts and methods of graph theory can be applied to a variety of real world problems.

Essential Questions:
- What are the primary terms of graph theory?
- What general principles can be justified in graph theory?
- How can the principles of graph theory be applied to real world problems?

Concepts:
- Graph theory terms and expressions
- Applications of graph theory
- Classic graph theory problems
- History of the development of graph theory

Competencies:
- Identify key terms in graph theory
- Determine whether a graph is traceable
- Explain and identify planar graphs
- Determine the chromatic number of a graph
- Apply the chromatic number of a graph to real world problems
- Form the adjacency matrix of a graph
- Explain the Four Color Map Theorem
- Determine the minimal spanning tree of a graph
- Apply Euler's Formula

Big Idea #2: The study of the structures of surfaces can reveal connections to graph theory principles and to real world applications.

Essential Questions:
- What is topology?
- How are surfaces considered topologically equivalent?
- How is topology related to graph theory?
- What is a torus?
- What is a Mobius Strip?
- What are the topological features of a Mobius Strip?
- What are the applications of Mobius Strips?
- What are minimal surfaces?
- How can minimal surfaces be demonstrated?

Concepts:
- Topological equivalence
- Torus
- Mobius Strip
- Klein Bottle
- Minimal surfaces
- Features of soap bubbles

Competencies:
- Determine whether shapes are topologically equivalent
- Identify a torus
- Analyze classic games played on a torus
- Explain the topologic features of a Mobius Strip
- Explain the applications of a Mobius Strip
- Explain the topologic features of a Klein Bottle
- Describe minimal surfaces
- Explain the topological features of soap bubbles
- Demonstrate minimal surfaces using soap bubbles

DELAWARE VALLEY SCHOOL DISTRICT

Curriculum Plan

Unit 1: Combinatorics

Time Range in Days: 30 Days

Overview: Students will use principles and processes of combinatorics to count the number of ways that an event can occur.

Focus Questions:

- How can we count the number of ways that an event can occur without counting one by one?
- How can the formulas for combinatorics be applied to real world problems?

Goals: Students will be able to

- Count without really counting
- Apply formulas and processes in combinatorics to real world problems
- Utilize a graphing calculator to assist in solving combinatorics problems

Objectives: Students will be able to

- Define combinatorics. (DOK Level One)
- Analyze various problems in combinatorics, including problems concerning the number of Friday the 13^{th}'s, number of colored blocks, and number of paths. (DOK Level Three)
- Identify and apply the Multiplication Principle of Counting. (DOK Level Two)
- Solve consecutive integer problems. (DOK Level Two)
- Evaluate the factorial of a positive integer. (DOK Level Two)
- Simplify expressions involving factorials. (DOK Level Two)
- Explain the formula for a permutation and use it to solve combinatorics problems. (DOK Level Three)
- Explain why 0!=1. (DOK Level Three)
- Solve problems involving permutations of items that are not all alike. (DOK Level Three)
- Explain the formula for a combination and use it to solve combinatorics problems. (DOK Level Three)
- Solve the number of possible paths on a grid problem. (DOK Level Three)
- Solve combinatorics problems in which items are ordered in a circle. (DOK Level Three)
- Solve combinatorics problems in which people are to be seated according to given restrictions. (DOK Level Three)
- Explain the inclusion-exclusion principle. (DOK Level Two)

- Apply the inclusion-exclusion principle to problems involving the number elements in a union of two sets. (DOK Level Three)
- Explain the extended inclusion-exclusion principle, and use it to solve combinatorics problems. (DOK Level Three)
- Use the inclusion-exclusion principle to find the number of derangements of a set of natural numbers. (DOK Level Four)
- Explain the pigeonhole principle. (DOK Level Two)
- Use the pigeonhole principle to solve combinatorics problems, including the Mutual Acquaintance/Stranger problem. (DOK Level Four)
- Use the pigeonhole principle to explain solutions to American Mathematics Competition problems. (DOK Level Four)
- Solve problems involving permutations of items that are not all alike. (DOK Level Three)
- Identify Pascal's Triangle. (DOK Level One)
- Explain the method of determining entries of Pascal's Triangle. (DOK Level Three)
- Use the table feature of a graphing calculator to find terms of Pascal's Triangle. (DOK Level Four)
- Find the sum of a sequence of numbers using the Sequence feature on a graphing calculator. (DOK Level Four)
- Apply Pascal's Triangle to the coefficients of terms in a binomial expansion. (DOK Level Four)
- Expand given binomials. (DOK Level Three)
- Explain the terms of the Fibonacci Sequence. (DOK Level Two)
- Relate the terms of Pascal's Triangle to the terms of the Fibonacci Sequence. (DOK Level Three)
- Solve the medical procedure problem. (DOK Level Four)
- Use the graphing calculator table feature to support the solution to the medical procedure problem. (DOK Level Three)
- Calculate the number of ways that 5 card poker hands can be formed from a standard deck of cards. (DOK Level Four)
- Use the combinatorics techniques learned to solve problems from previous American Mathematics Competitions. (DOK Level Four)

Core Activities:

- Direct instruction of an introduction to combinatorics using problems such as the number of Friday the 13^{th}'s, the number of ways to color a block, and the number of paths in a grid
- Student investigation and discovery of the formulas for the Multiplication Principle of Counting, permutations, and combinations
- Direct instruction on the simplifying of factorials

- Class discussion on the difference between a permutation and a combination
- Individual practice with applied problems such as the number of paths on a grid, number of ways to order items in a circle, and number of ways to seat people according to certain restrictions
- Class discussion on the Inclusion-Exclusion Principle
- Student investigation on the applications of the Inclusion-Exclusion Principle
- Direct instruction of derangements
- Student discovery the application of derangements using the problem of handing back quizzes in a random order
- Direct instruction on the Pigeonhole Principle
- Class discussion on the Mutual Stranger/Acquaintance Problem and its connection to the Pigeonhole Principle
- Guided practice on applying combinatorics principles to math contest problems
- Class discussion about Pascal's Triangle and its connection to the binary expansion
- Direct instruction on the use of a graphing calculator to find terms of Pascal's Triangle
- Class discussion on the connection of Pascal's Triangle with the Fibonacci Sequence
- Guided practice and class discussion on applying combinatorics principles to real world problems
- Apply the combinatorics techniques learned to solve problems from previous American Mathematics Competitions

Assessments:

Diagnostic:

- Teacher observation and questioning
- Pre Calculus Final Exam

Formative:

- Teacher observation and questioning
- Homework assignments
- Graded homework assignments
- Group assignments
- Quizzes

Summative: Common assessment for Unit 1

Extensions:

- Extra problems from the textbook
- Mathematics contest problems
- Teacher designed problems

Correctives:

- Re-teaching
- Teacher designed problems

Materials and Resources:

- Mathematics of Choice –How to Count Without Counting by Ivan Niven
- Mathematics contest problems (Pennsylvania Math League, American Mathematics Competition, Purple Comet Math Contest)
- Websites: www.combinatorics.net
  www.mathworld.wolfram.com/Combinatorics.html
  www.ptri1.tripod.com
  www. mathworld.wolfram.com/PascalsTriangle.html
  www.csam.montclair.edu/~kazimir/history.html

Unit 2: Probability

Time Range in Days: 30 Days

Overview: Students will use principles of probability to explain the counterintuitive results in applied problems.

Focus Questions:

- How can we solve complex probability problems?
- What probability problems have historical significance?
- How can we explain the counterintuitive results that often occur in probability?

Goals: Students will be able to

- Solve complex probability problems
- Explain the history of significant probability problems
- Analyze complex probability problems and explain their solutions

Objectives: Students will be able to

- Calculate and analyze the probability of 5 card poker hands selected from a standard deck of cards. (DOK Level Four)
- Analyze the Hat Color Problem. (DOK Level Four)
- Explain the Monty Hall problem. (DOK Level Three)
- Analyze the solution to the Monty Hall problem and use computer simulations to support the solution. (DOK Level Four)
- Analyze the Travelers’ Dilemma Problem. (DOK Level Four)
- Explain the drug testing problem. (DOK Level Three)
- Analyze the solution to the drug testing problem and use a graphing calculator to compare effects of changes in factors. (DOK Level Four)
- Explain the problem of minimizing the cost of drug testing. (DOK Level Three)
- Analyze possible solutions to the problem of minimizing the cost of drug testing, and explain the differences in these solutions. (DOK Level Four)
- Explain the concept of a Markov Chain. (DOK Level Two)
- Determine possible applications of Markov Chains. (DOK Level Two)
- Create the initial matrix and the transition matrix of a Markov Chain problem. (DOK Level Three)
- Apply Markov Chains to answer questions to various probability problems, including the Runner’s Shoes problem, the Cooties problem, the Lucky Charms problem, and the Playground problem. (DOK Level Four)
- Use graphing calculators and Derive software to assist in solving Markov Chain problems. (DOK Level Three)

- Explain the idea of expected value of a probability problem. (DOK Level Three)
- Use computer simulations (VisualBasic programs) to explain the expected value of the Cooties problem. (DOK Level Three)
- Use computer simulations (VisualBasic programs) to explain the idea of Random Walks. (DOK Level Three)
- Explain the idea of a skewed distribution, and apply it to the expected value of the Cooties problem, random walks, the theory of the disappearance of .400 baseball batting averages, and the theory of bacterial dominance. (DOK Level Four)
- Analyze the River Crossing Game. (DOK Level Three)
- Apply the combinatorics techniques learned to solve problems from previous American Mathematics Competitions. (DOK Level Three)

Core Activities:

- Student investigation on the number of poker hands in a standard deck of cards
- Class activity and discussion on the Hat Color Problem
- Class activity and discussion on the Monty Hall Problem
- Class activity and discussion on the Travelers' Dilemma Problem
- Direct instruction and class discussion on the Drug Testing Problem
- Direct instruction on the use of a graphing calculator to assist in describing the solution to the Drug Testing Problem
- Direct instruction and class discussion on Markov Chains using the Runner's Shoes Problem to motivate discussion
- Student investigation into the application of the Cooties Game to Markov Chains
- Independent practice on forming the transition matrix for the Cooties Game
- Guided practice using "Derive" software to assist in solving problems involving the Cooties Game
- Guided practice using computer simulations to explain the expected value of the Cooties Game
- Guided practice using computer simulations for random walks and explaining their connection to the Cooties Game
- Guided practice on the application of Markov Chains to the Lucky Charms game of "Swirls and Spoons"
- Direct instruction on the idea of a skewed distribution and its application to the Cooties Game, random walks, and the theory of the disappearance of .400 baseball batting averages, and the theory of bacterial dominance
- Class activity and discussion of the River Crossing Game
- Guided practice with using the probability techniques learned to solve problems from previous American Mathematics Competitions

Assessments:

Diagnostic:

- Teacher observation and questioning
- Combinatorics Common Assessment

Formative:

- Teacher observation and questioning
- Homework assignments
- Graded homework assignments
- Group assignments
- Quizzes

Summative: Common assessment for Unit 2

Extensions:

- Extra problems from the textbook
- Mathematics contest problems
- Teacher designed problems

Correctives:

- Re-teaching
- Teacher designed problems

Materials and Resources:

- www.mathworld.wolfram.com/MarkovChain.html
- Cooties Game
- Lucky Charms "Swirls and Spoons"
- Computer simulations (VisualBasic programs) of the Monty Hall problem
- Computer simulations (VisualBasic program) of the Cooties problem and Random Walks
- Introduction to Derive for Windows by Bernhard Kutzler
- "Why Mathematicians Now Care About Their Hair Color" article by Sara Robinson
- "Travelers' Dilemma" article by Kaushik Basu
- Full House by Stephen Jay Gould, p.147-151
- "The River Crossing Game" from Mathematics Magazine, February, 2007
- Mathematics contest problems (Pennsylvania Math League, American Mathematics Competition, Purple Comet Math Contest)
- Teacher designed worksheets

Unit 3: Applications of Matrices

Time Range in Days: 15 Days

Overview: Students will apply matrix arithmetic and algebra to real world problems.

Focus Questions:

- What are the rules of arithmetic for matrices?
- How can matrix equations be solved?
- How can matrices be used to solve applied problems?

Goals: Students will be able to

- Simplify matrix expressions
- Solve matrix equations
- Solve systems of equations using matrices
- Apply matrices to real world problems

Objectives: Students will be able to

- Add, subtract, and multiply matrices, by hand and with the use of a graphing calculator. (DOK Level Two)
- Identify valid properties of matrices. (DOK Level Two)
- Determine the inverse of a matrix using a graphing calculator. (DOK Level Two)
- Use the inverse of a matrix to solve algebraic equations. (DOK Level Three)
- Create the corresponding matrix to a system of linear equations. (DOK Level Two)
- Use a graphing calculator to determine the reduced row echelon form of a matrix, and interpret it as the solution to a system of linear equations. (DOK Level Three)
- Write the optimizing equation and constraint inequalities to a linear programming problem. (DOK Level Three)
- Draw the feasibility region of a linear programming problem and sample isoprofit lines, and use them to solve the problem. (DOK Level Four)
- Identify binding and non-binding constraints of linear programming problems. (DOK Level Three)
- Investigate and analyze sensitivity analysis for various factors involved in linear programming problems. (DOK Level Four)
- Explain the Simplex Algorithm for solving linear programming problems. (DOK Level Three)
- Use matrices to encode messages. (DOK Level Four)
- Use inverse matrices to decode messages. (DOK Level Four)

- Explain and apply methods of encrypting messages, including monoalphabetic ciphers, polyalphabetic ciphers, a Vigenere Square, and the Enigma Machine. (DOK Level Four)

Core Activities:

- Direct instruction on the arithmetic and algebra of matrices
- Direct instruction and guided practice of properties of matrices and using a graphing calculator to manipulate matrices
- Guided practice on solving systems of equations using matrices
- Direct instruction on linear programming models
- Guided practice with determining the feasible region of a linear program model and visualizing the isoprofit line to solve the problem
- Independent practice solving linear program models
- Direct instruction on operations research and the possible extensions of basic linear program models
- Student investigation and discovery of the use of matrices to encode and decode messages
- Direct instruction and class discussion on encryption methods, including monoalphabetic ciphers, polyalphabetic ciphers, a Vigenere Square, and the Enigma Machine

Assessments:

Diagnostic:

- Teacher observation and questioning

Formative:

- Teacher observation and questioning
- Homework assignments
- Graded homework assignments
- Group assignments
- Quizzes

Summative: Common assessment for Unit 3

Extensions:

- Videos from the "Numberphile" website
- Teacher designed problems

Correctives:

- Re-teaching
- Teacher designed problems

Materials and Resources:

- Cryptological Mathematics by Robert Edward Lewand
- Introduction to Operations Research by Frederick S. Hiller and Gerald J. Lieberman
- Precalculus by Michael Sullivan, p. 727-743, 767-773
- "Two Centuries On, a Cryptologist Cracks a Presidential Code" by Rachel Emma Silverman
- Websites: www.mathworld.wolfram.com/LinearProgramming.html
  www.mathworld.wolfram.com/Cryptography.html
  www.people.hofstra.edu/Stefan_Waner/RealWorld/Summary4.html

Unit 4: Graph Theory and Topology

Time Range in Days: 15 Days

Overview: Students will learn the basic terminology of graph theory and apply it to real world problems and to the fundamentals of topology.

Focus Questions:

- What are the basic terms of graph theory?
- What general principles can be formulated in graph theory?
- What is the historical development of graph theory?
- What are the most significant problems from graph theory?
- How can graph theory be applied to real world problems?
- What is topology?
- How are objects considered topologically equivalent?
- What are minimal surfaces and how can we analyze their structure?

Goals: Students will be able to

- Apply basic terms of graph theory
- Formulate and justify general principles of graph theory
- Explain the historical significance of graph theory
- Apply graph theory principles to real world problems
- Identify topologically equivalent objects
- Analyze minimal surfaces

Objectives: Students will be able to

- Identify and explain the concepts of vertex, edge, and adjacent. (DOK Level Two)
- Identify and explain graphs that are complete, bipartite, and complete bipartite, and use proper notation to represent such graphs. (DOK Level Three)
- Determine the degree of a vertex on a graph. (DOK Level One)
- Analyze the Sprouts game and strategies for winning the game. (DOK Level Four)
- Explain the Bridges of Königsberg problem. (DOK Level Three)
- Use the concept of degree to explain the concept of a traceable graph. (DOK Level Three)
- Identify and explain the chromatic number of a graph. (DOK Level Three)
- Determine the chromatic number of a given graph. (DOK Level Three)
- Write an adjacency matrix to represent a given graph. (DOK Level Three)
- Draw a graph given its adjacency matrix. (DOK Level Three)

- Use the concept of chromatic number of a graph in applications such as the exam scheduling problem and the chemical storage problem. (DOK Level Four)
- Explain and solve the Houses and Wells problem. (DOK Level Three)
- Explain the concept of a planar graph, and identify such graphs. (DOK Level Three)
- Determine how many colors are needed to color a given map. (DOK Level Three)
- Prove that any map consisting of straight lines is 2-colorable. (DOK Level Three)
- Explain the 4 Color Map Theorem and its history. (DOK Level Three)
- For a given region, use Euler's Formula to determine one of the following values given the other two: the number of vertices, number of edges, number of regions. (DOK Level Four)
- Relate Euler's Formula for graphs to Euler's Formula for polyhedra. (DOK Level Four)
- Determine whether a graph is connected, a cycle, or a tree. (DOK Level Two)
- Determine the minimal spanning tree of a weighted graph. (DOK Level Three)
- Use trees and weighted graphs to solve and explain applied problems such as the telephone company problem. (DOK Level Three)
- Describe how two figures are topologically equivalent. (DOK Level Two)
- Create a Möbius strip and analyze its properties. (DOK Level Three)
- Differentiate between the concepts of one-sided and two-sided. (DOK Level Three)
- Differentiate between a torus and various knots. (DOK Level Three)
- Analyze classic games played on a torus. (DOK Level Four)
- Describe a Klein Bottle. (DOK Level Three)
- Explain, in terms of topology and related to concepts from calculus, the reason why bubbles are round. (DOK Level Three)
- Discuss the concept of minimal surface and relate it to shapes formed by soap films. (DOK Level Three)
- Demonstrate minimal surfaces using soap films and variously shaped wires. (DOK Level Three)

Core Activities:

- Direct instruction on the basic terminology of graph theory
- Student investigation and class discussion into the properties of graphs, including complete graphs, bipartite graphs, traceable graphs, and chromatic number
- Direct instruction on the Bridges of Königsberg problem

- Guided practice applying the idea of chromatic number to real world problems
- Group activity of playing the Sprouts Game
- Guided practice in writing the adjacency matrix of a graph
- Student investigation and class discussion on the Houses and Wells problem
- Direct instruction on planar graphs and how to determine whether a graph is planar
- Student investigation and class discussion on the Four Color Map Theorem and its controversy
- Student investigation on Euler's Formula for the number of edges, vertices, and regions of a graph
- Guided practice with the ideas of connected, cycle, and tree
- Student investigation and class discussion on the minimal spanning tree of a weighted graph
- Direct instruction and class discussion on how two figures are considered topologically equivalent
- Student discovery and analysis of classic games played on a torus
- Student investigation and discovery of properties of Möbius Strips
- Direct instruction of the idea of a Klein Bottle
- Direct instruction on the idea of minimal surfaces
- Student investigation and discovery of properties of soap bubbles and surfaces created by them

Assessments:

Diagnostic:

- Teacher observation and questioning

Formative:

- Teacher observation and questioning
- Homework assignments
- Graded homework assignments
- Group assignments
- Quizzes

Summative: Common assessment for Unit 4

Extensions:

- Videos from the "Numberphile" website
- Teacher designed problems

Correctives:

- Re-teaching
- Teacher designed problems

Materials and Resources:

- Euler's Gem by David S. Richeson
- Graph Theory-Euler's Rich Legacy by Wayne Copes, et. al.
- The Mathematical Universe by William Dunham
- Möbius Story Wind and Mr. Ug [www.keepvid.com].mp4
- Websites: www.mathworld.wolfram.com/GraphTheory.html
  www.mathworld.wolfram.com/MoebiusStrip.html
  http://www.oglethorpe.edu/faculty/~j_nardo/2000-2001/Spring%202001/great%20ideas/torusgames/index.html
  www.bubbles.org
  www.virtualmathmuseum.org
  www.numberphile.com

Unit: Miscellaneous Advanced Mathematics Topics

Time Range in Days: If time allows

Overview: Students will solve and analyze solutions to various problems selected by the teacher as appropriate.

Core Activities:

- Solve challenging and non-routine problems.
- The teacher, based upon the students' interests and the teacher's fields of expertise, may select appropriate additional topics. Such topics may include statistics, linear algebra, abstract algebra, applications of calculus, mathematical modeling, and history of mathematics. Problems may be selected from challenging competitions such as the Pennsylvania Math League, American Mathematics Competition, and American Invitational Mathematics Exam.

Extensions: Extra problems from the textbook
Teacher designed problems

Correctives: Re-teaching
Teacher designed problems

Instructional Methods:

Direct Instruction
Teacher Questioning
Individual Practice
Group Discussion

Materials and Resources:

These problems may come from various sources, including previous mathematics contests such as the Pennsylvania Mathematics League, the American Mathematics Competition, and the American Invitational Mathematics Exam.

Assessments:

Diagnostic: Teacher Observation
Formative: Homework Assignments
Teacher Observation
Summative: Graded Homework
Quiz or Test